

# Multi-Objective Layout Optimization for Freight Comprehensive Transportation Networks

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**Abstract:** This paper studies the layout of freight comprehensive transportation networks (FCTNs) in order to reduce cost and increase efficiency. After reconstructing the real world FCTNs to be a new directed graph with multiple attributes for nodes and edges, a multi-objective layout optimization model is proposed. Numerical results based on the case of coal comprehensive transportation networks in Shanxi, China show the effectiveness of the proposed model.

## 1. Introduction

The freight comprehensive transportation networks (FCTNs) are multi-layer networks, composed of multiple modes of transportation, such as railways, highways, and waterways et al. With the rapid development of China's economy, the rational arrangement of the transportation lines and nodes in FCTNs plays a very important role in the logistics industry. The layout of FCTNs not only affects logistics cost, but also has a significant impact on logistics service level. In the context of reducing cost and increasing efficiency in the logistics industry, the study of the layout of FCTNs has important significance in theoretics and practices.

Recently, many results on the layout of logistics networks has been achieved in many areas, such as networks with hierarchical nodes, multi-modal hub and spoke logistics networks, railway cold chain logistical networks, networks of hierarchical emergency response facilities, pharmaceutical logistics intermodal networks, and supply chain et al. Liu et al<sup>[1]</sup>. develop a collaborative layout model for hierarchical nodes considering nodes and links capacity constraints. Wang et al<sup>[2]</sup>. establish an intermodal single-allocation p-hub logistics network model taking into account the congestion and capacity limit at the hub. Zhang et al<sup>[3]</sup>. construct a spatial layout model of the hub and spoke railway cold chain logistical network in China by the determination of the radiation range of each hub province with the improved gravity model. Paul et al<sup>[4]</sup>. analyse the effectiveness of the current and optimal locations of a set of existing regional assets by formulating a multi-objective hierarchical extension of the maximal covering location problem. Yang et al<sup>[5]</sup>. construct a model of location-routing-inventory optimization of highway, rail and air transport hubs with capacity limits by setting carbon emission, cost and time as targets. Zhang et al<sup>[6]</sup>. build a mixed integer non-linear program model to solve the problem of long logistics delivery time in supply chain. Han et al<sup>[7]</sup>. analyse the present situation, attributes and functions of Inner Mongolia road coal logistics nodes and determine the layout of multi-grade coal logistics nodes. Zhao<sup>[8]</sup> establishes a kind of hub-and-spoke regional logistics network based on hub-and-spoke model.

In this paper, we study the layout problem of FCTNs by balancing the cost and the efficiency. We first model the real world FCTNs to be a directed graph with multiple attributes for nodes and edges. Then a multi-objective layout optimization model is proposed to reduce cost and increase efficiency of FCTNs. Finally, numerical results on the coal transportation network in Shanxi, China are given to illustrate the effectiveness of the proposed model.

## 2. The FCTNs and Its Reconstructed Network

In this section, we first describe the real world FCTNs by graph theory, and then give a reconstructed network for the FCTNs.

### 2.1. The FCTNs

In the real world FCTNs, there are many kinds of nodes, such as demanders, providers, highway transport hubs, railway stations and ports et al. We denote them by  $v_i$ s, and the collection is denoted by  $V = \{v_1, v_2, \dots, v_n\}$ , where  $n$  is the number of nodes. The connections between nodes are transportation lines, such as railways, highways, waterways et al. In general, the travelling of goods from the origin to the destination, goes through different types of nodes and different transport ways. The connection from node  $v_i$  to node  $v_j$  is denoted by  $e_{ij}$ , and the collection is denoted by  $E = \{e_{ij} \mid \text{There is a connection from node } v_i \text{ to node } v_j, \text{ where } i \neq j\}$ .

In the layout of FCTNs, there are also some nodes and lines to be build. The collection of nodes and lines to be build are denoted by  $V_b$  and  $E_b$  respectively. The construction cost of transportation line  $e_{ij}$  is denoted by  $b_{ij}$ . And if the transportation line  $e_{ij}$  is already in existence, that is,  $e_{ij} \in E$ , then there is no construction cost and let  $b_{ij} = 0$ . The collection of  $b_{ij}$  is denoted by  $B$ . The construction cost of node  $v_i$  is denoted by  $\bar{b}_i$ . And if the node  $v_i$  is already in existence, that is,  $v_i \in V$ , then there is no construction cost and let  $\bar{b}_i = 0$ . The collection of  $\bar{b}_i$  is denoted by  $\bar{B}$ .

There are many attributes for transportation lines between the pair of nodes, such as mileage, passing capacity, shipping cost and delivery time et al. For every transportation line  $e_{ij} \in E \cup E_b$ , these attributes are all finite and nonnegative. The mileage, passing capacity, shipping cost(per unit, per mileage) and delivery time of transportation line  $e_{ij}$  are denoted by  $d_{ij}$ ,  $p_{ij}$ ,  $c_{ij}$  and  $t_{ij}$  respectively, and the collections are denoted by  $D$ ,  $P$ ,  $C$  and  $T$ .

The transshipment of goods from one transport way to another is taken place in nodes with these transport ways. There are also some attributes for nodes, such as transshipment capacity, transshipment cost and transshipment time et al. For node  $v_i \in V \cup V_b$ , the transshipment capacity, transshipment cost(per unit), transshipment time are all finite and nonnegative, and are denoted by  $\bar{p}_i$ ,  $\bar{c}_i$  and  $\bar{t}_i$  respectively, and the collections are denoted by  $\bar{P}$ ,  $\bar{C}$  and  $\bar{T}$ . If there is only one transport way between node  $v_i$  and other nodes, then there is no transshipment, and let  $\bar{p}_i = 0$ ,  $\bar{c}_i = 0$ ,  $\bar{t}_i = 0$ .

The demand and supply for goods in node  $v_i \in V \cup V_b$  is denoted by  $h_i$ .  $h_i > 0$  indicates the supply for goods in node  $v_i$ ,  $h_i < 0$  indicates the demand for goods in node  $v_i$ , and  $h_i = 0$  indicates no demand or supply for goods in node  $v_i$ . The collection of  $h_i$  is denoted by  $H$ .

Thus, we can describe the real world FCTNs as a directed graph  $G = (V \cup V_b, E \cup E_b, D, P, \bar{P}, C, \bar{C}, T, \bar{T}, B, \bar{B}, H)$ .

### 2.2. The Reconstructed Network for FCTNs

In order to reduce the complexity of layout optimization, we further give a reconstructed network for FCTNs.

For every node  $v_i \in V \cup V_b$ , let  $m$  be the number of transport ways between node  $v_i$  and other nodes in the FCTNs. If  $m > 1$ , that is, there is more than one transport way between node  $v_i$  and other nodes, then divide node  $v_i$  into  $m$  nodes  $v_{i_1}, v_{i_2}, \dots, v_{i_m}$ . For simplicity, we also denoted the node set by  $V \cup V_b$ . The connection from node  $v_{i_j}$  to  $v_{i_k}$  represents the transshipment from transport way  $j$  to transport way  $k$  in node  $v_i$ . The connections between node  $v_{i_j}$  and other nodes represents the connections between node  $v_i$  and other nodes by transport way  $j$  in the origin FCTNs. The collection of edges is also denoted by  $E \cup E_b$ .

The construction cost of connection from node  $v_{i_j}$  to  $v_{i_k}$  represents the construction cost of node  $v_i$  for the transshipment from transport way  $j$  to transport way  $k$ , and is denoted by  $b_{i_j i_k}$ . The transshipment capacity, transshipment cost, transshipment time of connection from node  $v_{i_j}$  to  $v_{i_k}$  represents the transshipment capacity, transshipment cost, transshipment time of node  $v_i$  in the origin

FCTNs, and are denoted by  $p_{ij}$ ,  $c_{ij}$  and  $t_{ij}$  respectively, and let  $d_{ij,i_k} = 1$ . The passing capacity, shipping cost and delivery time of connections between node  $v_{ij}$  and other nodes by transport way  $j$  are the same as those between node  $v_i$  and these nodes in the origin network.

For every node  $v_i$  in the origin network, if the demand and supply for goods  $h_i \neq 0$  and the number  $m > 1$  (the transport ways between node  $v_i$  and other nodes), then add a new node  $v_{i_{m+1}}$  in the new reconstructed network and let  $h_{i_{m+1}} = h_i$ . And add edges from node  $v_{i_{m+1}}$  to  $v_{i_k}$  ( $1 \leq k \leq m$ ) if  $h_i > 0$ , add edges from node  $v_{i_k}$  ( $1 \leq k \leq m$ ) to  $v_{i_{m+1}}$  if  $h_i < 0$ . And let  $h_{i_k} = 0$  for  $1 \leq k \leq m$ . The collection of  $h_i$  is also denoted by  $H$ .

Thus, the origin FCTNs can be reconstructed to be a new directed graph, which is also denoted by  $G = (V \cup V_b, E \cup E_b, D, P, C, T, B, H)$ .

### 3. Multi-Objective Layout Optimization Model for FCTNs

In this section, we propose a multi-objective layout optimization model for FCTNs by balancing cost and efficiency.

#### 3.1. The Cost Function

Economically, the advantages and disadvantages of network layout are measured by the total cost during goods flow in FCTNs. The cost includes the construction cost of transportation lines and nodes, shipping cost and transshipment cost.

For every edge  $e_{ij} \in E \cup E_b$ , let  $x_{ij}$  be the freight volume going through this edge and  $x = (x_{ij})$  be the matrix of freight volumes. Then the shipping cost or transshipment cost in edge  $e_{ij}$  is

$$C_{ij} = c_{ij}d_{ij}x_{ij} \quad (1)$$

Let  $I(\cdot)$  be the indicator function, that is,

$$I(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{else} \end{cases} \quad (2)$$

Then the construction cost of transportation line or node in edge  $e_{ij}$  is

$$B_{ij} = b_{ij}I(x_{ij}) \quad (3)$$

Thus, the total cost during goods flow in FCTNs is

$$f_1(x) = \sum_{e_{ij} \in E \cup E_b} c_{ij}d_{ij}x_{ij} + b_{ij}I(x_{ij}) \quad (4)$$

#### 3.2. The Efficiency Function

In terms of efficiency, the advantages and disadvantages of network layout are measured by the time turnover quantity during goods flow in FCTNs. The travelling time of goods from the origin to the destination includes delivery time and transshipment time. Thus, the efficiency indicator of goods travelling in FCTNs is defined by

$$f_2(x) = \sum_{e_{ij} \in E \cup E_b} t_{ij}x_{ij} \quad (5)$$

#### 3.3. The Multi-Objective Layout Optimization Model

The layout optimization of FCTNs is to decide the network structure under the existing traffic network, the network to be build, and the transportation tasks, such that the total cost is minimal and the efficiency is maximal. That is, which edges/nodes are chosen to be used or to be build. For every edge  $e_{ij} \in E \cup E_b$ , the freight volume  $x_{ij} > 0$  means this edge is chosen, otherwise  $x_{ij} = 0$  means this edge is not chosen. Thus, The multi-objective layout optimization model for FCTNs can be described as

$$\min f_1(x) = \sum_{e_{ij} \in E \cup E_b} c_{ij} d_{ij} x_{ij} + b_{ij} I(x_{ij}) \quad (6)$$

$$\min f_2(x) = \sum_{e_{ij} \in E \cup E_b} t_{ij} x_{ij} \quad (7)$$

$$\text{s.t.} \begin{cases} \sum_j x_{ij} - \sum_j x_{ji} = h_i, \quad \forall v_i \in V \cup V_b \\ 0 \leq x_{ij} \leq p_{ij}, \quad \forall e_{ij} \in E \cup E_b \end{cases} \quad (8)$$

$$\quad (9)$$

Where, Eq.(6) is one objective which minimizes the total cost, includes the construction cost of transportation lines/nodes and shipping/transshipment cost. Eq.(7) is another objective which minimizes the time turnover quantity, equivalently, maximizes the overall efficiency of goods travelling in FCTNs. Eq.(8) is the flow balance constraints, which mean that the difference of inflow quantity and outflow quantity in node  $v_i$  is  $h_i$ . Eq.(9) is the capacity constraints, which mean that the freight volume  $x_{ij}$  going through edge  $e_{ij}$  is nonnegative and is not more than the passing/transshipment capacity  $p_{ij}$ .

### 3.4. The Satisfactory Solution of the Model

Due to the contradiction between cost and efficiency, multi-objective optimization model Eq.(6)-(9) generally does not have optimal solutions and we can only find satisfactory solutions. In order to get a satisfactory solution, we can transform the multi-objective optimization model into a single-objective model by the linear weight method.

Let  $f_1^*$  and  $f_2^*$  be the optimal values of the single-objective models Eq.(6), (8), (9) and Eq.(7)-(9) respectively, and let  $\alpha$  and  $1 - \alpha$  (where  $\alpha \in [0,1]$ ) be the coefficients of weight for objective  $f_1(x)$  and  $f_2(x)$ . Then the multi-objective optimization model Eq.(6)-(9) can be transformed into the following single-objective model

$$\min f(x) = \alpha \frac{f_1(x)}{f_1^*} + (1 - \alpha) \frac{f_2(x)}{f_2^*} \quad (10)$$

s.t. Eq.(8) and (9)

Where Eq.(10) is the objective which minimizes the weighted average of the normalized total cost and the normalized time turnover quantity with the coefficient  $\alpha$ .

## 4. Numerical Results

In this section, numerical results are presented to illustrate the effectiveness of the proposed multi-objective layout optimization model Eq.(6)-(9) for FCTNs. Numerical experiments are conducted by Matlab based on the data set from the coal transportation network in Shanxi, China (CTNS), which is shown in<sup>[9]</sup>.

The main content of optimizing the layout of the coal transportation network includes the spatial layout of transportation lines and nodes, reasonable allocation of transportation capacity, transshipment capacity, and transportation ways. The flow distribution of the Shanxi coal transportation network are shown in Fig.1.

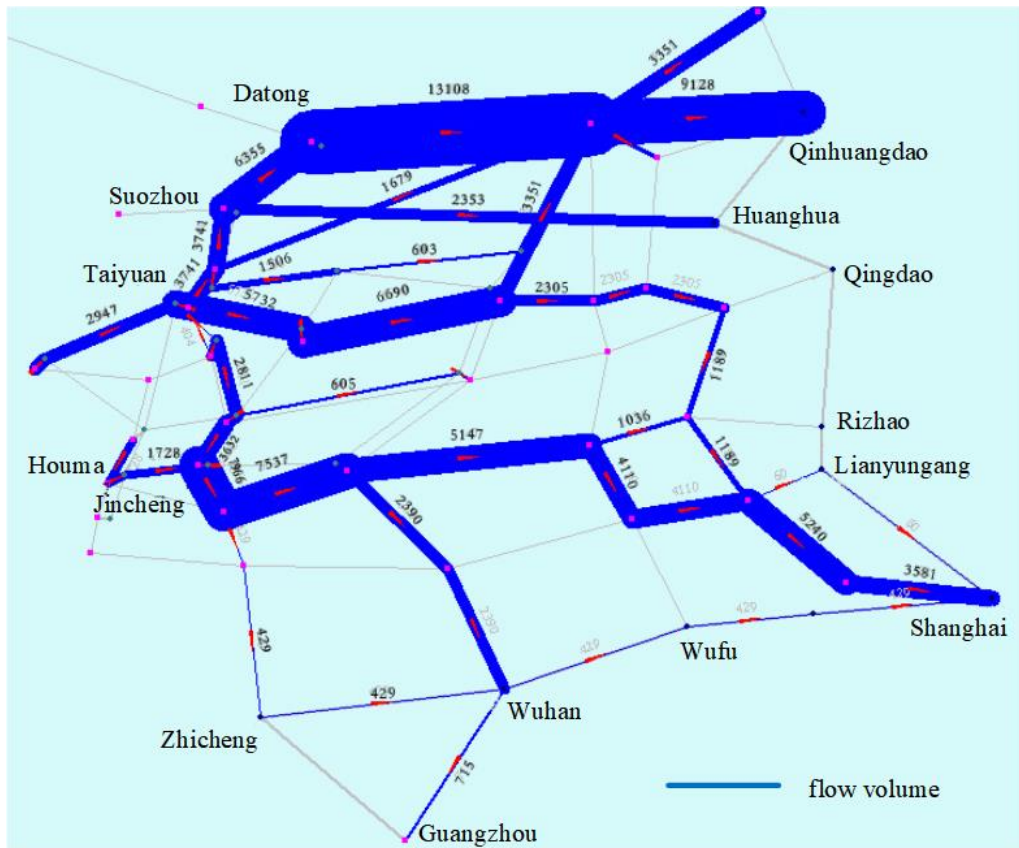


Figure 1 The flow distribution of the Shanxi coal transportation network.

From the results, it can be seen that the direction of coal transportation in Shanxi Province is roughly from west to east and from north to south. Coal transportation is mainly carried out by railways and supplemented by highways. Highway transportation is only for short distance consolidation, while railway transportation is the main form of long distance transportation. And because of the high cost of transshipment, it is generally not considered to transfer coal between different modes in long distance transportation. Only when the railway transportation capacity is insufficient, highway transportation is used for short distance transportation.

Due to the distribution characteristics of geographical location, using the city center as the source of coal may cause deviations from actual conditions. For example, the coal in Jincheng and Changzhi should be gathered in Houma and be transported by the Houyue Line in the actual geographical distribution. Due to limitations in data statistics, in the optimization plan, the Jinyue line is used for outbound transportation. Although the transportation distance is relatively short, but the transportation capacity of the Jinyue line is small, it need to use highways for transit and consolidation. This actually increases the transportation cost and time. However, these do not affect the rationality of the entire layout plan.

## 5. Summary

In this paper, we investigate the layout optimization problem of FCTNs by considering the objectives of reducing cost and increasing efficiency. The real world FCTNs is first reconstructed to be a directed graph by the graph theory. Based on the flow distribution and the selection of transportation lines and nodes, a multi-objective layout optimization model is presented and a single-objective optimization model is proposed to get a satisfactory solution by the linear weight method. Numerical results on the coal transportation network in Shanxi, China are further given to show the effectiveness of the proposed model.

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